

The Rigour of Proof

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For the pluralist, proofs serve several purposes

Proofs guarantee truth *within* a theory.

Proofs guarantee truth *of* a theory from the perspective of a meta-theory.

Proofs guarantee the preservation knowledge, given what is thought to be already known, or is taken to be known.

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But now consider: for some theorems there are several non-equivalent proofs – equivalent in the result, but not in the methodology/ approach.

The pluralist accounts for this by saying that proofs also help us to *understand* mathematics through the careful work of proving.

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What is common to the above is that they are all self-justifying or can be traced to something that is self-justifying. Self-justification is justification in terms of meaning.

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The pluralist thinks of meaning as *relative to* a formal context and language. It is ordinal not cardinal. No further justification is forthcoming. It has reached a temporary stability that contributes to understanding. It follows that meaning changes over time and people.

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Pluralism	Mathematics consists in a plurality of foundations, methodologies, theories, applications and background philosophies	Proofs serve several purposes	Rigour comes in degrees and can be appropriate or inappropriate, 'appropriateness' is a normative judgement

Thank-you for your attention.

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